Name:

AP Calculus AB
PRACTICE Examination 1 - Solutions

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The examination contains ten problems which are worth 10 points each, and two bonus problems worth ten points each. All answers must be justified. An appropriate amount of work must be shown to receive credit.

Problem 1. (Sets)
Let $A = [1, 8]$ and $B = (5, 11]$. Find the indicated set.
Express your answer in interval notation.

Solution. Expressed in interval notation:

(a) $A \cup B = [1, 11]$

(b) $A \cap B = (5, 8]$

(c) $A \setminus B = [1, 5]$

(d) $B \setminus A = (8, 11]$

□

Problem 2. (Domain and Range)
Find the domain and range of $f(x) = \sqrt{x^2 - 1}$.

Solution. If $x \in \text{dom}(f)$, then $x^2 - 1 \geq 0$, so $x^2 \geq 1$, so $|x| \geq 1$. Thus $\text{dom}(f) = (-\infty, -1] \cup [1, \infty)$.
Now square root is always nonnegative; also, $f(x) \approx x$ for large x. Thus $\text{range}(f) = [0, \infty)$. □

Problem 3. (Absolute Value Inequalities)
Solve the inequality $|2x + 8| \leq 20$. Correctly write the solution set using interval notation.

Solution. Compute

$$\begin{aligned} |2x+8| \leq 20 \quad &\Leftrightarrow \quad -20 \leq 2x + 8 \leq 20 \\ &\Leftrightarrow \quad -28 \leq 2x \leq 12 \\ &\Leftrightarrow \quad -14 \leq x \leq 6 \end{aligned}$$

Thus, the solution set is $[-14, 6]$. □

Problem 4. (Quadratic Inequalities)
Solve the inequality $x^2 - 2x - 15 \leq 0$. Correctly write the solution set using interval notation.

Solution. Let $f(x) = x^2 - 2x - 15$. We wish to solve $f(x) \leq 0$. That is, we wish to find where the graph of f is below the x-axis. Factoring gives $f(x) = (x + 3)(x - 5)$. This is negative for x between -3 and 5; that is, the solution set is $[-3, 5]$. □

Problem 5. (Equation of a Line)
Consider the points $A = (-2, 9)$ and $B = (12, 2)$.
Find the slope-intercept form of the equation of the line through A and B.

Solution. Follow three steps:

Step 1. $m = \dfrac{\Delta y}{\Delta x} = \dfrac{2-9}{12-(-2)} = \dfrac{-7}{14} = -\dfrac{1}{2}$

Step 2. $y = m(x - x_0) + y_0 = \dfrac{1}{2}(x-12) + 2$

Step 3. $y = \dfrac{1}{2}x - 6 + 2$, so $y = \dfrac{1}{2}x - 4$

□

Problem 6. (Equation of a Circle)
Find the center and radius of a circle with equation

$$x^2 + 12x + y^2 - 14y = 21.$$

Solution. The general equation is $(x-x_0)^2 + (y-y_0)^2 = r^2$, where (x_0, y_0) is the center and r is a the radius. Complete the square to get

$$(x^2 + 12x + 36) + (y^2 - 14y + 49) = 21 + 36 + 49 = 106.$$

Now factor this to get

$$(x+6)^2 + (y-7)^2 = 106.$$

Thus the center is $(6, 7)$ and the radius is $\sqrt{106}$. □

Problem 7. (Wrapping Function)
Let $W : \mathbb{R} \to \mathbb{R}^2$ be the wrapping function. Find $W\left(\dfrac{53\pi}{6}\right)$.

Solution. We have

$$W\left(\frac{53\pi}{6}\right) = W\left(\frac{48\pi}{6} + \frac{5\pi}{6}\right) = W\left(4 \cdot 2\pi + \frac{5\pi}{6}\right) = W\left(\frac{5\pi}{6}\right) = W(150°).$$

The reference angle is 30° in the second quadrant, so the x-coordinate is negative and the y-coordinate is positive. Thus

$$W\left(\frac{53\pi}{6}\right) = \Big(-\cos(30°), \sin(30°)\Big) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right).$$

□

Problem 8. (Trigonometric Values)
If $\sin\theta = \dfrac{5}{13}$, what is $\cos\theta$?

Solution. The Pythagorean Identity gives

$$\cos(\theta) = \sqrt{1 - \sin^2(\theta)} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}.$$

□

Problem 9. (Average Rate of Change)
Let $f(x) = x^2 - 3x + 1$. Find the average rate of change of f on the interval $[1, 3]$.

Solution. The average rate of change of f on the closed interval $[a, b]$ is

$$m = \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} = \frac{1 + 1}{2} = 1.$$

$\square$

Problem 10. (Remainder Theorem)
Let $f(x) = x^4 - 9x^3 + 17x - 11$. Find $f(7)$.

Solution. Use synthetic division:

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7   |     1  -9  17 -11
    |
             7 -14  21
    -------------------
          1  -2   3  10
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We see that $f(7) = 10$. $\square$